1. It takes Kim twice as long to run 2,255 decimeters as it takes Cara to run 1/10 of a mile. They start 1 kilometer apart and begin running toward each other. How far, to the nearest meter, will Kim have run when they meet? Assume that 1 mile = 1.61 kilometers.
A. 357 m  
B. 412 m  
C. 467 m  
D. 489 m  
E. 588 m

2. Replace each letter in \( \text{ONE} + \text{ONE} = \text{TWO} \) with a base-10 digit so that identical letters are replaced by identical digits and different letters are replaced with different digits, \( T \) is the only odd digit, and \( O \) cannot be zero. What is the value of \( N? \)
A. 0  
B. 2  
C. 4  
D. 6  
E. 8

3. Which of the following numbers has the greatest value?
A. \( 2^{1000} \)  
B. \( 6^{500} \)  
C. \( 30^{200} \)  
D. \( 50^{100} \)  
E. \( 1000^{75} \)

4. The equation \( a^2 + b^2 + c^5 = 2019 \) has exactly one solution where \( a, b, \) and \( c \) are positive integers with \( a > b \). Find \( a + b + c \) for this solution.
A. 56  
B. 57  
C. 58  
D. 59  
E. 60

5. Let \( M \) be the smallest positive integer that has a remainder of 2 when divided by 3 and has a remainder of 4 when divided by 5. Let \( N \) be the smallest positive integer that has a remainder of 6 when divided by 7 and has a remainder of 8 when divided by 9. Find \( M + N \).
A. 70  
B. 72  
C. 74  
D. 76  
E. 78

6. There are 200 closed lockers numbered 1-200 in a locker room. Student 1 goes in and opens each locker, then student 2 goes in and closes every other locker (2, 4, 6, 8...). Student 3 then changes the state (opens it if it is closed, closes it if it is open) of every third locker (3, 6, 9, 12...), then student 4 does the same for every fourth locker (4, 8, 12, 16...). This continues until 200 students have gone through, with the \( n \)th student changing the state of all lockers numbered with a multiple of \( n \). How many lockers are open at the end?
A. 14  
B. 24  
C. 48  
D. 64  
E. 100

7. In some contexts, a function is defined to be linear if for all elements \( x \) and \( y \) in the domain and for all real numbers \( a, f(ax) = af(x) \) and \( f(x + y) = f(x) + f(y) \). Using this definition of linear, how many of the following functions from \( \mathbb{R} \) to \( \mathbb{R} \) are linear?
A. \( f_1(x) = 3x \)  
B. \( f_2(x) = 3x + 2 \)  
C. \( f_3(x) = 2 \)  
D. \( f_4(x) = 0 \)

8. Find the sum of all base-10 eight-digit (the first digit cannot be zero) numbers that contain no digits other than 0 or 1 (for example: 10100101, 10000000, 11111111).
A. 711,111,104  
B. 1,010,101,010  
C. 1,031,111,104  
D. 1,351,111,104  
E. 1,422,222,208

9. Find \( |a - b| \) if \( a \) and \( b \) are the two real solutions to \( (f \circ f \circ f)(x) = 1 \) for \( f(x) = 2x^2 + 28x + 91 \).
A. \( \sqrt{2}/4 \)  
B. \( \sqrt{2}/2 \)  
C. \( \sqrt{2}/2 \)  
D. \( \sqrt{2} \)  
E. \( \sqrt{2} \)

10. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says, “I am a knight.” Y says, “X is telling the truth.” Z says, “I am a spy.” Which of the following correctly identifies all three people?
A. X is the spy.  
B. X is the spy.  
C. X is the knight.  
D. X is the knight  
E. X is the knave.  
F. Y is the knight.  
G. Y is the knave.  
H. Z is the knave.  
I. Z is the spy.  
J. Z is the spy.
11. Let $M$ be the sum of the solutions to $e^{-x} \sin x - e^{-x} \cos x = 0$, where $0 \leq x < 2\pi$. Find $\csc M$.
A. -1  B. $\frac{-\sqrt{3}}{2}$  C. 1  D. $\frac{2}{\sqrt{3}}$  E. 2

12. Consider the following function in polar coordinates: $r = 2 + 0.5 \cos \theta$. Which of the following best describes the graph of this function?
A. Line  B. Two Lines  C. Hyperbola  D. Parabola  E. Ellipse

13. A biased die is rolled until two 1s are rolled in succession, or until a 1 and then a 2 are rolled in succession (in that order). The die lands on 1 with probability 50%, on 2 with probability 20%, and on something else with probability 30%. What is the probability that the rolling will end with successive 1s?
A. 1/3  B. 1/2  C. 4/7  D. 2/3  E. 5/7

14. Find the sum of all complex (both real and nonreal) zeros of $f(x) = \frac{x^3 - \frac{1}{2}}{x - \frac{1}{2}}$.
A. $-\frac{1}{2} - i \frac{\sqrt{3}}{2}$  B. $-\frac{1}{2}$  C. 0  D. $\frac{1}{2}$  E. $\frac{1}{2} + i \frac{\sqrt{3}}{2}$

15. How many ordered lists $(a, b, c, d, e, f)$ of nonnegative integers satisfy $a + b + c + d + e + f = 12$?
A. 5304  B. 5544  C. 6160  D. 6188  E. 6468

16. In parallelogram $ABCD$, $BC$ is extended beyond point $C$ to point $E$. Points $F$ and $G$ are the points of intersection of $AE$ with $BD$ and $CD$, respectively. If $FG = 12$ and $EG = 15$, then find $AF$.
A. 16  B. 18  C. 20  D. 24  E. 27

17. If the graphs of the functions $f(x) = b(x - m)^2 + n$ and $g(x) = x - m$ intersect, then what is the greatest possible value of $bn$?
A. 1/4  B. 1/2  C. 3/4  D. 1  E. 2

18. At a school, 69% of Math Club members are also in the Physics Club, and 79% of Math Club members are also on the Quiz Team. Consider the percentage, $P$, of Math Club members who are both in the Physics Club and also on the Quiz Team. Based on the given data alone, we can find a percentage $M$ and a percentage $N$ that will guarantee that $M \leq P \leq N$. What is the sum of the largest possible value for $M$ and the smallest possible value for $N$?
A. 52%  B. 90%  C. 117%  D. 127%  E. 148%

19. Some children are playing a game that uses a regular octagon $ABCDEFGH$. There are pennies on some of the sides: 1 on $\overline{AB}$, $\overline{BC}$, and $\overline{EF}$; 3 on $\overline{CD}$; 2 on $\overline{DE}$; and none on $\overline{FG}$, $\overline{GH}$, and $\overline{HA}$. Each child, in turn, may add a penny to each of two adjacent sides (for example, a child may add a penny to $\overline{AB}$ and a penny to $\overline{BC}$), but no other changes are permitted. Their goal is to reach a state where all sides have the same number of pennies. If $S$ is the smallest number of turns needed, which inequality does $S$ satisfy?
A. This is impossible  B. $S \leq 8$  C. $8 < S \leq 15$  D. $15 < S \leq 25$  E. $25 < S$

20. Five distinct integers $a, b, c, d, e$ are to be ordered from least to greatest. You are told that $e, d, c, b, a$ has at least 3 of the 5 values correctly placed; $e, b, c, d, a$ has an odd number of the values correctly placed; and $a, d, c, b, e$ is not the solution. You can choose 3 letters and learn their order from least to greatest. Which 3 should you choose to guarantee that the ordering of all 5 numbers can be correctly determined?
A. $a, b, d$  B. $a, b, e$  C. $b, c, d$  D. $b, c, e$  E. $c, d, e$